Order in Cognitive Systems: Constructal Theory, Neurocircuitry and the Extended Mind

Abstract

Research suggests that order in cognitive systems in best captured by principles of self-organization and field dynamics. We outline a recently-conceived principle of self-organized efficient flow, known as Constructal Theory, describing order common to biological, conceptual and engineered phenomena. Constructal patterns are observable in neural structure, in cortical activity, and in manifestations of the extended mind, such as language and databases. We hypothesize that intelligence is itself related to cognitive efficiency, wherein optimized efficiency is characterized as Constructal. We thereby create a testable conceptual model of efficient order in both the brain and the extended mind. We argue for the co-dependency of cognitive performance and efficient flow-like structures within relevant intentional artifacts.

Keywords: Extended mind; Principle of Least Effort; Zipf’s Law; Constructal Theory; Intelligence; Neuroanatomy.

What is the cause of *order or pattern* in cognitive systems? With the goal of advancing a testable model of 'ordering mechanisms' in human neurocircuitry, and in exemplars of the extended mind (Clark & Chalmers, 1998; Logan, 2007), we develop, in this paper, the notion that minds create a context of intentional artifacts (e.g. languages, databases, the mess of papers around your desk) that are structurally optimized for kinds of *flow*. This flow structure can described in terms of mathematical 'Principles of Least Effort', or efficiency, such as Zipf's Law (Zipf, 1949), and Constructal Theory (Bejan, 1997; Bejan & Lorente, 2010). The efficiency of cognition is similarly flow-dependent. We hypothesize that more efficient Constructal flow leads to better cognitive performance.

We examine data on the structure of neural activity, along with data on the structure of the extended mind. Due to its central role in cognition, we also address the structure of language. Analysis highlights that each of these cases shows tree-like flow properties that we define in terms of Constructal Theory (Bejan, 1997; Bejan & Lorente, 2010). This theory allows us to build a model of efficient flow in extended cognition.

We characterize our contribution to research in three ways. First, we create a conceptual model of efficient 'order' in the extended mind. We thereby argue for the co-dependency of cognitive performance and efficient flow-like structures within relevant intentional artifacts. Second, we introduce new applications of Constructal Theory to the domain of cognitive science. Third, we add to the growing literature that develops an externalist, non-individualist understanding of cognition and-action. Put alternatively, we proffer a conception of the mind in which it partially and reflexively *externalizes itself* in the form of efficient Constructal flow systems. Technology and language can be seen as flowing, vascular artifacts of shared, interactive cognitive externalization.

In the following section we develop our theoretical stance. We address desiderata for a theory or model of psychological order, with joint consideration of the extended mind. We then discuss the literature on Zipf's Law and Constructal Theory. Subsequently, we apply Constructal analyses to human neurocircuitry and to exemplars of the extended mind. The paper concludes with discussion of a unified model of 'efficiency' in the extended mind, proposing a novel hypothesis for a cognitive system's *intelligence*.

The Search for Order in Cognitive Systems

Order in cognitive systems has been traditionally conceived as the result of mediating endogenous entities, mechanisms, or processes such as representation, programs, or computation (Wagman, 2010). Phenomena such as development, movement, perception, or social behaviors, have been explained in terms of genetic programs, mental representations or brain-area specializations.

Yet "the emergence of order in physical or biological systems is typically explained by means of self-organization and field dynamics" (Wagman, 2010, p. 46). Examples of self-organization and field dynamics include models of birds' flocking behavior (Reynolds, 1987), or power laws to describe animals' movement (Pennycuick, 1975). Characteristic of these examples is that patterns, "emerge…from a cascade of dynamic local interaction across the various levels of a complex system…The global pattern itself is not contained within any of the local interactions but rather emerges as a lawful consequence of those interactions" (Wagman, 2010, p. 32). Furthermore, theories based on self-organization and field dynamics lay claim to parsimonious explanatory power (Wagman, 2010).

There is an apparent theoretical incommensurability between the cognitive on the one-hand, and the physical or biological on the other. Given the *situatedness* of cognition, we argue this incommensurability is a brake on advancing cognitive science. A theory of cognitive order profits from ontology shared with its physical and biological context, wherein shared ontology need not imply reduction to other sciences.

The Extended Mind and a Unifying Ontology

The ‘extended mind’ refers to artifacts in the external environment being employed by the mind such that they might be considered extensions to the mind itself (Clark & Chalmers, 1998). On this view, extracranial objects or symbols – the diarized aide-memoire, for example, or mental arithmetic accounted on one’s fingers - play a functional role in cognition. The mind and its external environment act as a dynamically coupled system. The mind is thereby extended. The focal criterion for determining an extension is that, via coupling, the artifacts in question have functional equivalence to the corresponding internal processes.

Language is a chief way in which cognitive processes find extension into the world (Clark & Chalmers, 1998). Instances of the extended mind are as numerous as couplings; discussions in this field of cognitive science have encompassed, for example, not just language but the entirety of culture (Logan, 2007). The scope of extensions is therefore broad and deep; broad because it includes language and culture (and more), and deep because human cognition may be, some cognitive scientists argue (e.g. Fodor, 1975), language-like at a fundamental level. With the understanding that the mind is ineluctably and interactively embedded in its physical and biological context, a model of order in cognitive systems should take *joint* account of intracranial cognition *and* the extracranial context in which it occurs.

Ordering Mechanisms: Two Principles of Least Effort

We began this paper with recent arguments for the virtue in using self-organization and field dynamics theory to explain order or pattern in cognitive systems. We followed with the view that a model of order must take joint account of the extended mind. Yet, as we describe throughout the remainder of this paper, each of the cited artifacts of the extended mind is patterned in some way. We now develop the notion that there is a *common* patterning mechanism, based on specific principles of self-organization and field dynamics, underlying these exemplars of the extended mind. The 'Principle of Least Effort' (Zipf, 1949) is discussed first. We subsequently generalize to the more recently-conceived Constructal Theory (Bejan, 1997).

The development of language is a remarkable outcome of human evolution. Some features of language suggest that it has been shaped by patterning forces. Resurgent in the research literature, and best known, is Zipf's Law.

Zipf's law states that, in natural language, the frequency of a word is inversely proportional to its rank in a frequency table. Hence, the most frequent word in a text occurs twice as often as the second most frequent word, which occurs twice as often as the fourth most frequent word, and so on.

Zipf's Law, in mathematical form, states:

$$\log R = a - b \log c$$

R is the rank ordering of the artifact (e.g. word) under consideration, c is the count of its appearances in a (suitably lengthy) text, and a and b are constants.

For Zipf's Law to hold in its original form then $b = 1$. In studies of language and other relevant datasets, such as city sizes, income levels, or corporate sizes, the value of b derived is typically within a few percentage points of one.

Some scholars regard Zipf’s Law as semantically vacuous. It has been shown, for example, that randomly-generated texts produce Zipfian distributions (Li, 1992). However, natural language approximates more closely to Zipf’s Law than does randomly-generated text (e.g. Losee, 2001). Furthermore, there are arguments for the law based on the observed function and use of language. It has been proposed to arise because both speaker and listener minimize effort in reaching understanding; an even distribution of effort leads to the observed distribution (Ferrer i Cancho & Sole, 2003).

Zipf’s Law thereby originates in the description of structure within natural language; a structure captured by a ‘Principle of Least Effort’. In contrast, Constructal Theory offers a physical law of self-organization for systems that flow in time (Bejan, 1997; Bejan & Lorente, 2010). While its disciplinary origin is thermal dynamics, it derives from the observation that if a flow system - such as a river basin or migratory path for animals - has sufficient freedom to shift its configuration, so it tends progressively to provide easier access to its flow.

Constructal law states, “for a finite-size flow system to persist in time (to live) it must evolve in time such that it provides easier access to the currents that flow through it” (Bejan & Merkx, 2007, p.2). The law identifies an overall pattern – rather than a precisely-deterministic rule of structure – that freely-adapting flow systems manifest. Constructal phenomena are commonly analyzed as problems of flow-maximization (or, comparably, flow-time minimization), wherein the flow can be from volume to point, area to point, line to point, or their respective reversals. We now outline, from first principles, a solution to this class of problems, producing a dendritic or tree-like pattern or 'design' of flow (Bejan & Merkx, 2007, provides a full derivation of these mathematical results).

Constructal design can be abstracted to a simple two dimensional context. Each point P in an area A is to be connected to an end-point M via the quickest route. A simple solution is to directly connect every P to M. This gives a radial structure. Given that a radial solution could only apply in the case of uniform travel times in all directions, a more realistic condition is that there is more than one mode of communication; perhaps one being significantly faster than the other. One might imagine that in, say, the historical development of towns and their modes of transport, settlements evolved footpaths that flowed perpendicularly into tracks for speedier horse-drawn wagons. Travel time is minimized if a group can move to the faster mode as soon as possible.

Assuming a rectangular area A_1, of length L_1 and height H_1, and two speeds of travel, we can now generalize the problem of flow-time minimization to: what is the optimal aspect ratio of A_1 with respect to time of travel? A group, say, can travel here at two speeds towards M; a low speed V_0 off the main 'route', and a much higher speed V_1 on it. Further assume that travelers are evenly spread throughout A_1 (see Fig. 1).

The average travel time of the population is (note that the integration with respect to *dy* begins at zero rather than $-H_1/2$ because the average travel time to M is identical on either side of the x-axis):

$$t_1 avg = \frac{1}{H_1 L_1} \int_0^{H_{1/2}} \int_0^{L_1} \left(\frac{x}{V_1} + \frac{y}{V_0} \right) dx\, dy$$

Which gives: $t_1avg = \frac{L_1}{2V_1} + \frac{H_1}{4V_0}$

The area $A_1=H_1L_1$ is fixed, and so

$$t_1avg = \frac{A_1}{2H_1V_1} + \frac{H_1}{4V_0}$$

By differentiating with respect to H_1 we find the minimum for travel time:

$$H_{1,min=\langle 2\frac{V_0A_1}{V_1}\rangle}1/2$$

Similarly;

$$L_{1,min=\langle \frac{V_1A_1}{2V_0}\rangle}1/2$$

And therefore, the optimal ratio of H_1 to L_1 is:

$$\left(\frac{H_1}{L_1}\right)_{opt} = \frac{2V_0}{V_1} < 1$$

The maximal travel time is from one of the distant corners ($x=L_1$ and $y=\pm H_1/2$) to the origin M. It is given by:

$$t_{1,}max = \frac{L}{V_1} + \frac{H_1}{2V_0}$$

The minimizations of time for the furthest-distance of travel and the average distance both elicit the same optimized shape. In other words, what is best for the worst-positioned point of flow (or traveler) is also best for average flow. Constructal design optimizes both worst-case and global flow. In the case of traveling agents, what is best for the self-interested individual is also best for the whole community.

By minimizing t_1, we find that:

$$t_{1,min=\langle 2\frac{A_1}{VoV_1}\rangle}^{1/2}$$

This leads to the conclusion that travel time should be partitioned equally on the slow and fast routes. This two-speed analysis can be iterated – to areas within this area, for example, with yet slower routes - to provide more intricate analyses, with higher branching order. Moreover, the assumption of perpendicular routes can be dropped to further increase realism (see Fig. 2).

Now, when the assumption of perpendicularity of routes is relaxed, we find that

$$t_1max = \frac{L}{V_1} + \frac{H_1}{2V_0}(\frac{1}{\cos\beta} - \frac{V_0}{V_1}\frac{\sin\beta}{\cos\beta})$$

The optimum angle for minimizing travel time is:

$$\beta opt = \sin^{-1}\frac{V_0}{V_1}$$

The equation for optimal angle β shows that when V_0 is much less than V_1 then β tends to zero; the pattern tends to perpendicularity. The minimal travel time is:

$$t_{1,min=\langle 2\frac{A_1}{V_0V_1}\cos\beta_{opt}\rangle}{}^{1/2}$$

Where speed increases more rapidly on the “main” route, there are longer, more slender designs. Where the increase in pace is steadier as one progresses to the “main” route, so the overall (e.g. street) plan is broader.

Note that unless speeds change by large factors, the angle β plays a minor role in optimization (here, the minimization of flow time). This leads to the insight that some visually-notable variations in the design-shape actually have trivial effect on system-wide flows. This, in turn, suggests that naturally occurring flow systems harbor many such ‘imperfections’, thereby explaining some of the challenge scientists have faced in theorizing order in natural flow systems (Bejan and Merkx, 2007).

Further deductions can be made (Heitor Reis, 2007 provides a derivation wherein the global constraint is energy and resistance to flow); optimal flow structure is a flow-tree in which the number of branches of each speed is proportional to their respective resistivity to flow. The average branch length varies inversely with resistivity, thereby producing a vascular structure with a large number of short, high-resistance branches, and a small number of long, low-resistance branches.

Constructal designs need not conform to a rectangular plan. We now consider another vascular design. In Fig. 3, it can be observed how branch lengths diminish with distance from the origin M. Imagine a group of ten travelers leaving M together, all with the same speed on each type of path. In common with the scheme described above, the central path is fastest, with (identical) declines in speed on each branch. The special property of this design is that the journey time from M to each end-point can now be identical. The travelers depart M together, and synchronously arrive at their respective branch end-points, E_i. We return to this 'pyramidal' design later.

Examples of biological Constructal flow include the migratory paths of animals, the circulatory and nervous systems, the lungs, and, of course, much from the botanical world. Constructal information processing can be in seen in language, the structuring of information in databases, and the flow of information between universities (Bejan, 2009). Constructal physical designs include street plans, the spatial distribution of settlement sizes, river patterns, thermal flow across metals, and lightning (Bejan & Merkx, 2007). In this way, the Constructal Law applies to biological, conceptual, and physical systems.

Zipf's Law can be interpreted in terms of Constructal Theory. The statistical comparison has been made in the context of settlement sizes and ranks (see Bejan, Lorente, Miguel & Reis, 2006, for an analysis of how towns evolve as a result of the efficient flow of goods and services between them and the surrounding communities, approximating to Zipf's Law for the frequency of settlement sizes). More generally, games' decision-trees, languages or complex databases can be viewed as vascular structures (e.g. Blasius & Toenjes, 2007), with hierarchical self-similarity, and central, commonly-used features (strategies, words, searches) that interconnect more peripheral elements. Indeed, Zipfian distributions are implied by self-similar hierarchical structures (e.g. Semboloni, 2008).

Few conditions are necessary for Constructal Law to apply. There must be, as stated above, freedom for the configuration of flow to adapt. Furthermore, there must be significant volume of flow. For example, we can analyze in Constructal terms of the migratory paths of thousands of wildebeest across the veldt, but not a dozen. Moreover, there must be 'memory' in the system;

paths trodden through the brush, trammeled and made sustainably easier to travel, perhaps. And this point is connected to the final condition; there must be 'interstitial spaces', i.e. zones of easier and harder flow. In other words, the area or volume inhabited by the flow system cannot be permanently uniform in resistance to flow.

In sum, Constructal Theory – another 'Principle of Least Effort' - offers a physical law describing order in flow systems. Constructal Theory creates a link from the physical to information processing, and from the natural to the human-engineered. It thereby aids joint analysis of cognition and the extended mind.

Neuroanatomy as Constructal

We now relate the ordering mechanism defined by Constructal Theory to human neuroanatomy. We address two exemplars of brain-bound Constructal flow. First, we describe neuron cell structures. Second, we examine hubs of activity in the cortex. Later, we discuss recent research on the correlation of higher intelligence with more efficient neurocircuitry.

Pyramidal cells are considered due to their critical role in cortical information processing (Spruston, 2008); analyses demonstrate that the length of the dendrites is proportional to branching order, as predicted by Constructal Theory. Pyramidal cells also have been linked to coordinating 'phasic' neural activity (Mann, Radcliffe & Paulsen, 2005). Areas that are 'in phase' in their electrical spiking activity are recruited within the same functional circuits. Some researchers have proposed that this is the basis of the 'attention binding' problem (Engel & Singer, 2001). Yet, as we noted in the discussion of Figure 3, the pyramidal design has the special property of optimizing flow wherein 'arrival times' are synchronous. Pyramidal neurons thus correspond to Constructal optimization under the phasic constraint.

We now move onto the second case of Constructal flow in the brain; the recent research on aggregated patterns of electrical activity between cortical ‘hubs’ of activity (Hagman et al., 2008). Here, hubs are considered “tied” by electrical activity flowing between them. More activity represents a stronger tie. Again, the number of ties is inversely proportional to tie strength; cortical activity shows dendritic form, with well-connected central hubs and weaker, less-connected peripheral areas.

In both cases of neural organization, we observe Constructal design; the number of branches is inversely proportional to size, and the branching pattern is consistent with optimized flow. At both the neuron-level and at the aggregated level of hubs of cortical activity, there is Constructal patterning.

Intelligence as Constructal: Two Hypotheses

Constructal properties of the brain, and the environment in which it is situated, have implications for our understanding of *intelligence*. Recent research points to *efficiency* in the distribution of neural activity as, in part, constituting higher intelligence (Gualtieri, 2002; Li et al., 2009). Given that optimally efficient flow distribution can be viewed in Constructal terms, we now reconsider extant research in the light of our model of efficient flow.

Li et al., (2009) hypothesize that higher intelligence derives from higher global efficiency of the brain’s anatomical network. They find that general intelligence scores are significantly correlated with network properties; shorter path lengths and higher overall efficiency. There is thereby support for the hypothesis that efficiency in neural architecture is a foundation of intelligence. More generally, reaction speed is positively correlated with intelligence for complex tasks (Schweizer, 2001), while withered (slower acting) neurons are associated with reduced intelligence (e.g. Comery et al., 1997).

Taking the research relating cognitive speed and efficiency to intelligence and combining it with a Constructal interpretation, we propose two hypothesizes:

H1. General intelligence (G) is positively correlated with more efficient Constructal flow in the brain's anatomical network.

H2. Extended mind intelligence depends on the efficiency of the extended flow structures in which the brain is functionally embedded. The more embedded in task-relevant Constructal flows an individual is, the more intelligent they are.

Discussion

We have endeavored to fulfill some of the promise of a model of cognitive order based on self-organization and field dynamics. Order is described as according with principles of efficiency; we focus on the dynamical model proffered by Constructal Theory. This view of vascular flow-systems has joint descriptive power for both human neuroanatomy and the extended mind, including languages. The coupling of the mind and its external context describes ways in which they are dynamically co-dependent.

We now address the limitations of the Constructal view. There is formidable evidence that biological designs, such as neuroanatomy, offer robustness against physical damage (Glassman, 1987). What would be called recurrence in neurophysiology, redundant ties in social networks, or redundant signals in information theory (e.g. Kolen & Kremer, 2001), can be ways in which networked systems are made robust, albeit at some cost to efficiency. We speculate that the brain is designed – in evolutionary terms – for some tradeoff between efficiency and robustness. So, while Constructal Theory is apt for modeling efficient flow, efficiency itself does not fully explain evolutionary fitness. We further note that Constructal Theory concerns persistence over time, and is less applicable to ephemeral processes. In this way, we suggest that Constructal models are better correlated with innate, automated and well-learnt cognition.

We capped our overall argument by hypothesizing that intelligence is Constructal. Insofar as efficient cognition is formative of intelligence, and insofar as that efficient cognition can be interpreted as a flow (to or from point or line, to area or volume), then it is optimized by Constructal design. In this sense, to be intelligent is to be efficient. To be efficient is to be Constructal. We further hypothesize that more intelligent subjects are not just more Constructal in their internal design; they are also able to create more efficient Constructal flows within the extended mind. The intelligence of the extended mind is re-framed in terms of embedding intracranial Constructal intelligence within the context; to creating vascular, goal-directed order.

Constructal Theory integrates diverse literatures on cognition and the extended mind. But Principles of Least Effort are surely not all that is behind cognition; intelligence, for example, implies choice, implying (some) wasted effort. And as we have noted, biological systems are perhaps best understood as comprising robustness-via-redundancy, rather than pure efficiency. However, we propose that elucidating a principle together with a testable mechanism that is common to neurocircuitry, its products and its context will assist future inquiries into cognitive order and capabilities.

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Figure 1. Travel speeds on two routes. (Adapted from Bejan & Merkx, 2007).

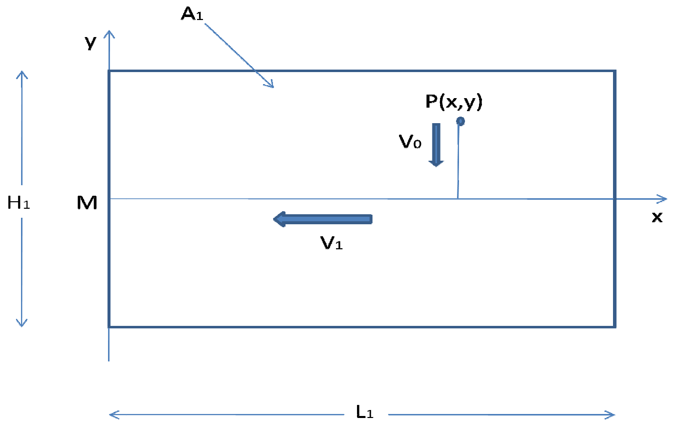

Figure 2: Angled routes. (Adapted from Bejan & Merkx, 2007).

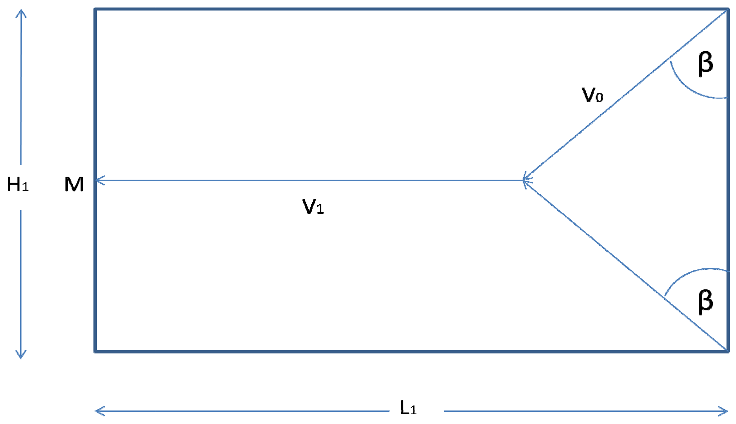

Figure 3. A pyramidal design. (Adapted from Bejan & Merkx, 2007).